"Modal analysis of an optical wave-guide having curvilinear square- shape cross-section."

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Abstract: In this contribution the modal analysis of a new type of non-conventional optical waveguide having curvilinear square shape cross-section is carried out. Dispersion curves of the waveguide are also obtained. The characteristic equations have been derived by using Goell's point matching method (GPMM) under weak-guidance approximation. The dispersion curves are also interpreted in two different cases. It has been observed that dielectric waveguide has more number of modes in comparison to metallic waveguide.

Key words: Optical waveguide, Dispersion curve, Modal analysis, curvilinear square shape cross-section.

Introduction

Much research work has done during the last forty years in the field of optical fiber technology. This leads to high capacity and high transmission rate system [1-8].Wave guide of unusual structure have generated great interest in comparison of conventional structure of optical wave-guides[9-14]. The materials used for production of optical waveguides like dielectrics, metals chiral, liquid crystal, polymers etc., have also played great role in revolutionized the communication technology[15-18].

In this paper an optical wave-guide having curvilinear square shape cross-section is proposed. This structure is generated by embedding two symmetrical inverted cardioids. This proposed wave guide is analyzed for two different cases. In one case, all boundaries of proposed wave-guide are taken as surrounded by dielectric material, while in other case they are taken as surrounded by conducting material. For both cases modal characteristic equations and corresponding dispersion curves are obtained by using Goell's point matching method (GPMM) [19].

When two symmetrical inverted cardioids are embedded in a common cladding, the shape may appear as shown in Figure (1). Waves are guided inside cardioids core and proposed structure is a part of cladding. The theoretical study in this region is carried out by us and it may be compared by experimental findings. The dielectric wave-guides are fundamental building block of integrated optics. They are not only used as transmission medium but also as components: filters, and directional couplers. Therefore modal analysis of such system is very important.

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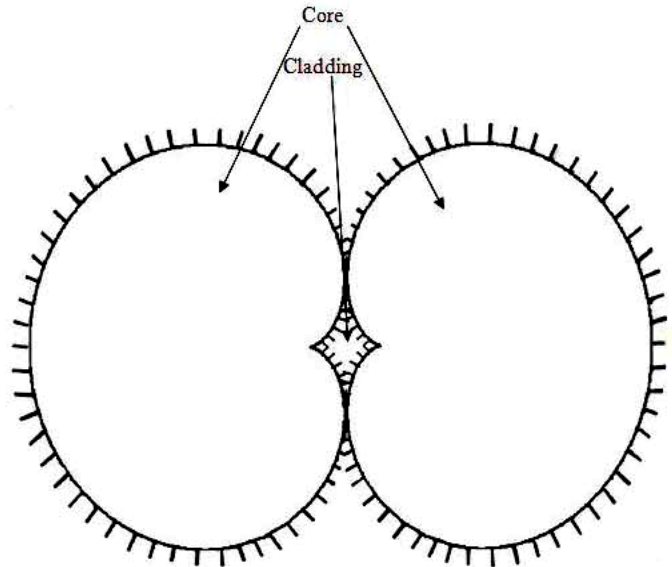**nigam.kni@gmail.com,spsingh.kni@gmail.com**

Figure.1: **Two symmetrical inverted cardioids embedded in a common cladding to generate proposed structure.**

Theory

For a wave guide with core and cladding refractive indices n_1 and n_2. $n_1 - n_2/n_1$ Can take as smaller than one under weak guidance approximation.

To study the proposed optical wave guide Goell's point matching method (GPMM) is employed. For this scalar wave equation in cylindrical polar co-ordinate(r,θ, z) system may be written as

$$\nabla^2 \psi - \frac{n^2}{c^2}\frac{\partial^2 \psi}{\partial^2 t} = 0 \qquad (1)$$

Here n stands for refractive index of the core or cladding region as the case may be and the function ψ stands for

the z-component of the electric field (E_z) or magnetic field (H_z). Considering harmonic variation of ψ with t and z, it can be written as

$\psi = \psi_0 \exp j(\omega t - \beta z)$. Now equation (1) takes the form

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + \left(\frac{n^2 \omega^2}{c^2} - \beta^2\right)\psi = 0 \qquad (2)$$

Here, ω, β and c are Optical angular frequencies z- Component of the propagation vector, and velocity of light in free space respectively. If ε and μ are permittivity of and permeability of the medium respectively then equation (2) can be written as

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + \sigma^2 \psi = 0$$

$$(3)$$

Such that $\sigma^2 = \omega^2 \mu \varepsilon - \beta^2$

To use separation of variables technique function ψ can be given as

$$\psi(r,\theta,z,t) = f_1(r)f_2(\theta)\exp j(\omega t - \beta z)$$

$$(4)$$

With this expression equation (3), can be written as

$$\frac{\partial^2 f_1(r)}{\partial r^2} + \frac{1}{r}\frac{\partial f_1(r)}{\partial r}\left(\sigma^2 - \frac{v^2}{r^2}\right)f_1(r) = 0$$

$$(5)$$

And $\dfrac{\partial^2 f_1(\theta)}{\partial \theta^2} + v^2 f_2(\theta) = 0$ \qquad (6)

here v is a non-negative integer.

The solution of equation (5) in terms of Bessel function and the solution of equation (6) in term of trigonometric functions like $\cos(v\theta)$ or $\sin(v\theta)$ can be found. The solution of the core as well as the cladding region can be taken as the linear combination of product of Bessel's functions and trigonometric functions of various orders. In cladding region, $\left(\sigma^2 - \dfrac{v^2}{r^2}\right) < 0$ and in guiding region, $\left(\omega^2 \mu \varepsilon - \beta^2\right) > 0$ The solution can be taken as the sum of a series where each term is the product of Bessel's function of the first kind $J_v(x)$ and trigonometric function of same order where as in the outer non-guiding region, the solution can be taken as linear combination of the product of modified Bessel's functions of second kind $K_v(x)$ and the trigonometric functions.

If ψ_1 and ψ_2 represents the solution in the core and cladding regions respectively, then we have

$$\psi_1 = A_0 J_0(u\,r) + A_1 J_1(u\,r)\cos\theta + B_1 J_1(u\,r)\sin\theta$$
$$+ A_2 J_2(u\,r)\cos 2\theta + B_2 J_2(u\,r)\sin 2\theta + \ldots\ldots\ldots(7)$$

and

$$\psi_2 = C_0 K_0(wr) + C_1 K_1(wr)\cos\theta + D_1 K_1(wr)\sin\theta$$
$$+ C_2 K_2(wr)\cos 2\theta + D_2 K_2(wr)\sin 2\theta + \ldots\ldots(8)$$

The parameters 'r' and 'θ' in the above equations (7) and (8) represents the polar-coordinates of the various points on the boundary of optical wave guide. The parameters 'u' and 'w' is defined as

$$u^2 = \left(\frac{2\pi\, n_1}{\lambda}\right)^2 - \beta^2, \text{ core - region parameter.} \qquad (9)$$

$$w^2 = \beta^2 - \left(\frac{2\pi\, n_2}{\lambda}\right), \text{cladding - region parameter.} (10)$$

In Goell's point-matching method, the fields in core region and cladding region are matched at selective points on boundary of the wave guide. In this case eighty points are taken on the core-cladding boundary to obtain reasonable results. Matching the field along with their derivatives at chosen points on the boundary of the wave-guide in each case following equations are found.

$$\sum_{q=0}^{39} A_q J_q(ur_k)\cos(q\theta_k) + \sum_{q=0}^{39} B_q J_q(ur_k)\sin(q\theta_k)$$

$$-\sum_{q=0}^{39} C_q K_q(wr_k)\cos(q\theta_k) - \sum D_q K_q(wr_k)\sin q\theta_k = 0$$

(11)

For   k=1, 2, 3……..80.

$$u\left[\sum_{q=0}^{39} A_q J'_q(ur_k)\cos q\theta_k + \sum_{q=0}^{39} B_q J'_q(ur_k)\sin q\theta_k\right]$$

$$-w\left[\sum_{q=0}^{39} C_q K'_q(wr_k)\cos q\theta_k + \sum_{q=0}^{39} D_q K'_q(wr_k)\sin q\theta_k\right]$$

For   k= 1, 2, 3……..80.   (12)

The prime terms in the above equation represents differentiation with respect to arguments and quantities r_k and θ_k are the polar co-ordinates of the core-cladding boundary. After taking the summation explicitly, a set of 160 simultaneous linear equations involving the constants A_q, B_q, C_q and D_q are obtained .These coefficients form a 160×160 determinant Δ_I .A non–trivial solution may exist for these set of equations

$$\Delta_I = 0 \qquad\qquad (13)$$

Equation (13) is the characteristic equation, which contains all the information about the modal properties of proposed waveguides. The solution of equation (13) gives propagation constants β for sustained modes in core region. In both cases, as shown in figure (2) and in figure (3), dispersion curve are obtained and interpreted. The equation (13) gives the normalized propagation constant.

$$b = \frac{\left(\dfrac{\beta^2}{k^2} - n_2^{\ 2}\right)}{(n_1^{\ 2} - n_2^{\ 2})} \qquad (14)$$
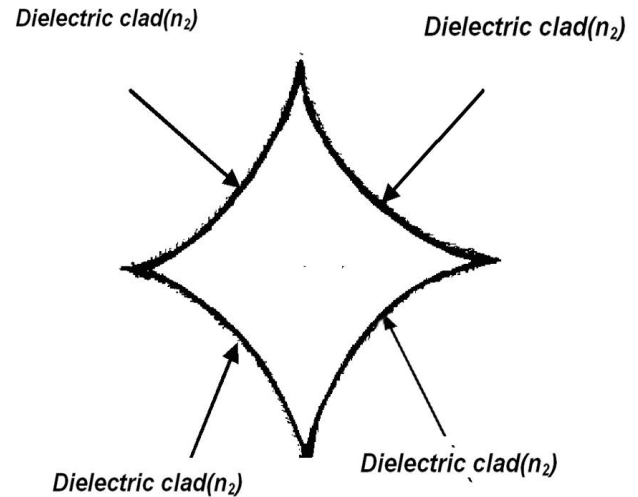

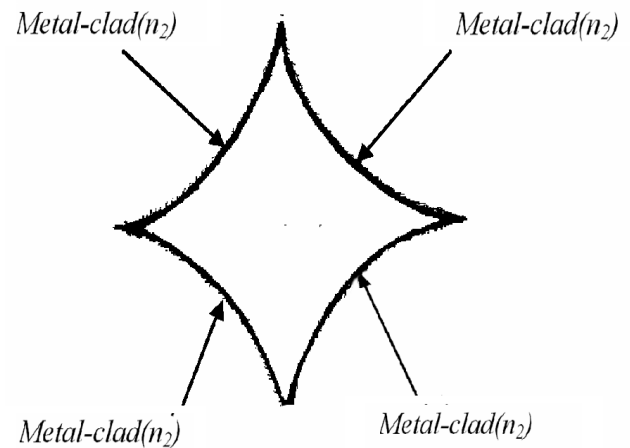**Figure.2**:Cross-sectional view of dielectric curvilinear square core optical wave guide.

Figure.3:Cross-sectional view of metal loaded curvilinear square optical wave guide.

For first few guided modes for proposed waveguide. The wave-guide parameter V can be written as,

$$V = \left(\frac{2\pi}{\lambda_0}\right) d \sqrt{n_1^2 - n_2^2} \qquad (15) \qquad .l$$

Similar analysis can be carried out for the case, when a conducting material has surrounded the entire boundary in place of dielectric. In this case, all boundaries are metallic; fields on all four sides must vanish. Therefore derivative part has no significance, which results in 80 simultaneous equations involving 80 unknown constants. These equation are converted to an 80×80 determinant $=\backslash \Delta_{II}$ consisting of constants A_q, B_q, C_q and D_q. For existence of non-trivial solution

$$\Delta_{II} = 0 \qquad (16)$$

The solution of equation (16) gives normalized propagation constants for a few guided modes.

Results and Discussion: To understand physical consequences of characteristic equations (13) and (16), the dispersion curves of sustained modes have to obtain. For refractive index of core (n_1) and cladding (n_2) are taken as 1·48 and 1·46 respectively. The wave length λ_0 of light in free space is taken as 1·55 μm .For each case, the left hand side of corresponding characteristic equation is plotted against the admissible values of β ($k_0 n_1 \geq \beta \geq k_0 n_2$) for fixed value of d, and zero crossings are noted. The zero crossing value corresponds to a particular sustained mode. Several such curves are plotted for different values of d for a given mode. The quantity d is related to wave-guide parameter and we have,

$$V = \frac{2\pi}{\lambda_0} d \sqrt{n_1^2 - n_2^2} \ .$$

With the help of V , we can calculate the normalized propagation constants 'b' using equation (14). In this way, b-V curves (dispersion curves) can be plotted for each mode and for each case. The same procedure is used for characteristics equation (16) for obtaining the b-V curves.

Figure-4 shows dispersion curves (b-V) for that case when a boundary of proposed waveguide is composed of dielectric material i.e. a dielectric optical wave-guide. While, figure-5 shows the dispersion curves for that case, when the boundaries of waveguide is composed of conducting material i.e. a metallic optical wave-guide. These obtained curves have the standard shape. However, there are many possible modes, but we have shown only six modes for dielectric optical wave-guide (DOWG) and first two modes for metallic optical wave-guide (MOWG).
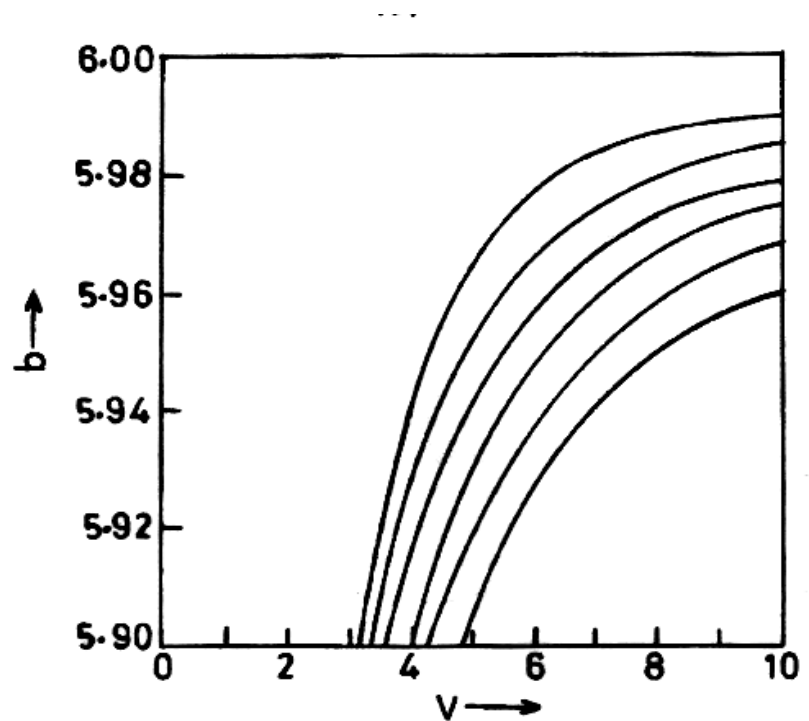

Figuire.4. Dispersion curves for DOWG.

Here, we have obtained some important and interesting preliminary insights, which are as follows:

1. The dielectric wave guide (DOWG) and the metallic optical waveguide (MOWG) shows the mode bunching properties near the cut-off value. In the case of standard square dielectric optical waveguide normally all modes lie between V= 0 · 0 and V= 1·70 .Whereas in our proposed wave-guide all the modes lie between V=3.12 and V=4.80.

2. It has been found that first cut-off value shifts from

V=0.0 to V=3.12 and the sixth cut-off value shifts from V=1.70 to V= 4.80 approximately. The distortion of standard square shape into curvilinear square shape is also responsible for an increase in the number of sustained modes for V=4.80(say). This means that more power will be transmitted through proposed optical wave-guide than a standard square wave-guide with common value of d.
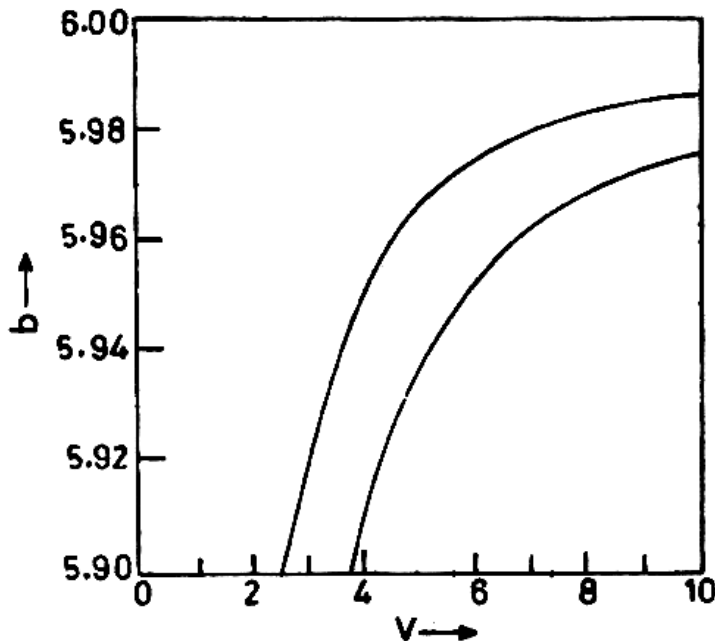

Figure.5: **Dispersion curves for MOWG.**

3. Near the cut-off value for the lowest order modes for standard square wave-guide, the effect of curvilinear square distortion is to shifts to the cut- off value slights towards a larger value of V.

 4.In case of dielectric optical wave guide (DOWG), the cut-off values V are crowded in the region=3.12-4.80, where as in the case of metallic wave guide it is separated and reduced which is an interesting feature and is also expected.

Conclusion: It can be concluded that proposed DOWG are useful for transmitting more power. The DOWG are more sensitive to small distortion in shape. The metallic waveguide can be used as mode filter while DOWG can be used for switching action in optical system.

The authors hope that the predicted results will be of sufficient interest to induce researchers worldwide to take up the experimental verification of the results in the present communication

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